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Frank B. Sachse

Computational Cardiology

Modeling of Anatomy, Electrophysiology, and Mechanics

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Preface

Biomedical research is at a critical point at present. The research has led to an enormous amount of data and models describing these data, but approaches for application, formalization and integration of this knowledge from the molecular to the system level are still topics of ongoing research and certainly far from fully developed.

Also in cardiology the different anatomical and physiological constituents as well as the coupling between them are being researched in increasing detail and are often described using computer-based models. But for this domain an integrative framework is still missing.

The application of computer-based modeling as a research, development and clinical tool often necessitates the coupling of various models from different levels. Describing the interactions between these models, which are both physically sound and computationally efficient, determines the applicability of such promising computer-based attempts.

My hope is that this book will contribute to the comprehension, spread and impact of computer-based modeling in cardiology, both from a teaching point of view and by summarizing knowledge from several, commonly delimited topics relating to the cardiac manifoldness.

The book evolved from revision and extension of my professorial dissertation (Habilitationsschrift) "Mathematical Modeling of the Mammalian Heart" written in 2002. This dissertation was based on notes for the lectures "Computational Biology: Bioelectromagnetism and Biomechanics," "Simulation of Physical Fields in the Human Body," and "Anatomical, Physical and Functional Models of the Human Body," which I gave at the Universität Karlsruhe (TH) from 1998 to 2003.

Salt Lake City, 1 February 2004 Frank B. Sachse

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